# Influence of specimen thickness on transport characteristics of water diffusion in black locust wood

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# Theoretical background

**1.Fick's law and diffusion equation:**

$$q = -D\frac{\partial c}{\partial x}$$

q – intensity of water flux
$\partial c/\partial x$ –gradient of concentration
D – diffusion coefficient

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

t - time

**Potential of transport:**

$c=\rho_0.w$

w – moisture content
$\rho_0$ – oven dry density

# The particular solution of the diffusion equation

1. Constant initial condition.
2. Symmetry of moisture content distribution, zero water flux at the center of the specimen.
3. Boundary condition in the form:

$$D \left. \frac{\partial c}{\partial x} \right|_{x=s} = \alpha \left( c_r - c(s,t) \right)$$

s - half thickness of the specimen
$\alpha$ - the surface emission coefficient
$c_r$ - the equilibrium concentration

$$\frac{c(x,t)-c_o}{c_r-c_o}=1-2\sum_{n=1}^{\infty}\frac{\cos\left(\delta_n\frac{x}{a}\right)Bi}{\left(\delta_n^2+Bi^2+Bi\right)\cos\delta_n}e^{-\delta_n^2 Fo}$$

$\delta_n$ are nonnegative roots of the equation $\delta tg\delta=Bi$ and Bi is the Biot number which is defined as:

$$Bi=\frac{\alpha s}{D}$$

Number Fo is the Fourier number:

$$Fo=\frac{Dt}{s^2}$$

# Material and method

*Robinia pseudoaccacia, L.* - 2% habitat in Slovakia

Specimens with dimensions 5 x thickness x 5 $cm^3$ in anatomical directions (R x T x L)

Thicknesses approximately between 5 and 16mm .

The oven dry specimens were allowed to absorb water vapour of moist air above aqueous saturated salt solution NaCl at 20°C temperature.

Average concentration:

$$\bar{c}(t) = \frac{1}{s}\int_0^s c(x,t)\,dx$$

$$\frac{w}{w_r} = 1 - \sum_{n=1}^{\infty} \frac{2Bi^2}{\delta_n^2\left(\delta_n^2 + Bi^2 + Bi\right)} e^{-\delta_n^2 Fo}$$

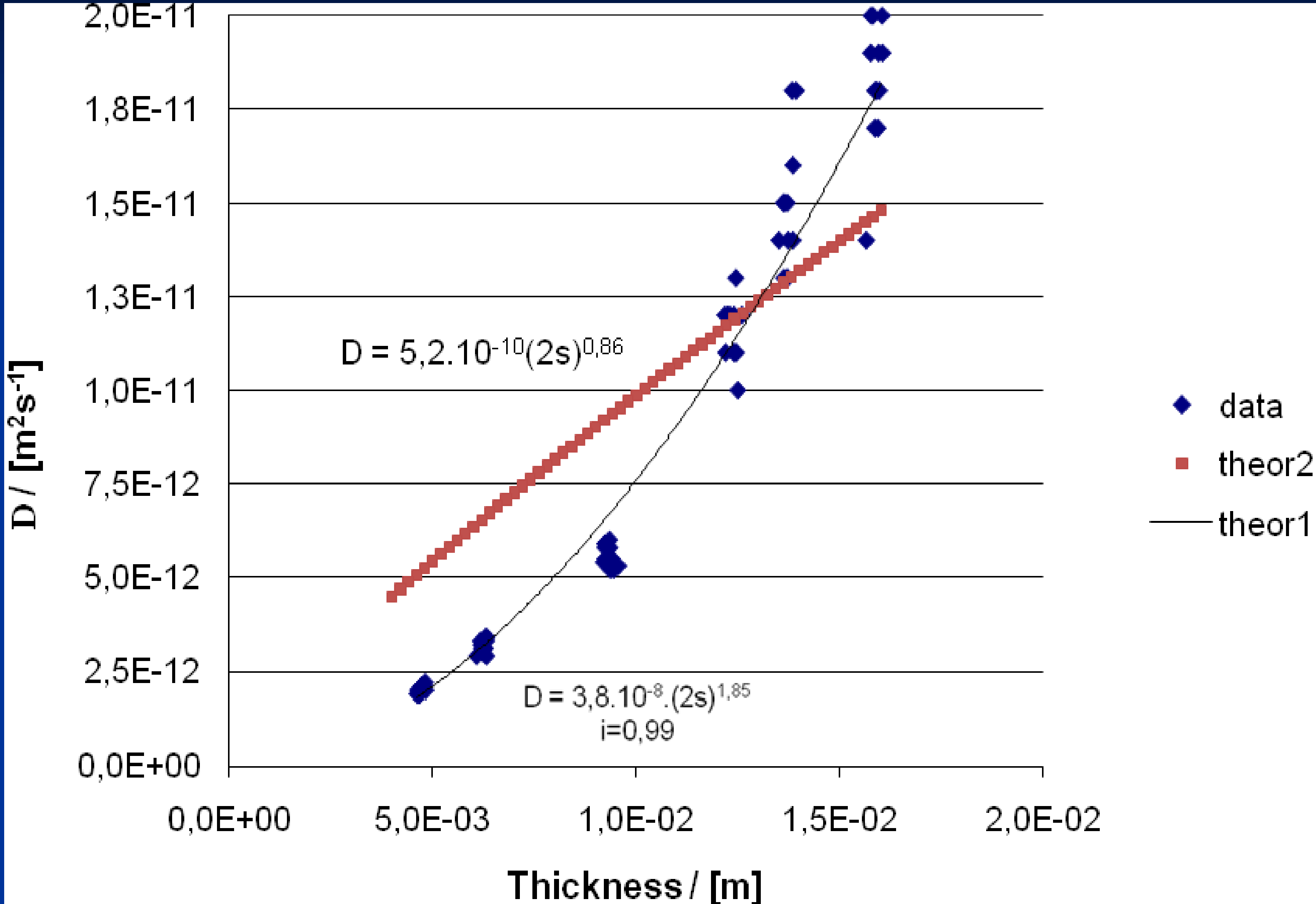
Least square method:

$$Q(D,\alpha) = \sum_{i=1}^{n} (w_{teori} - w_{\exp i})^2$$

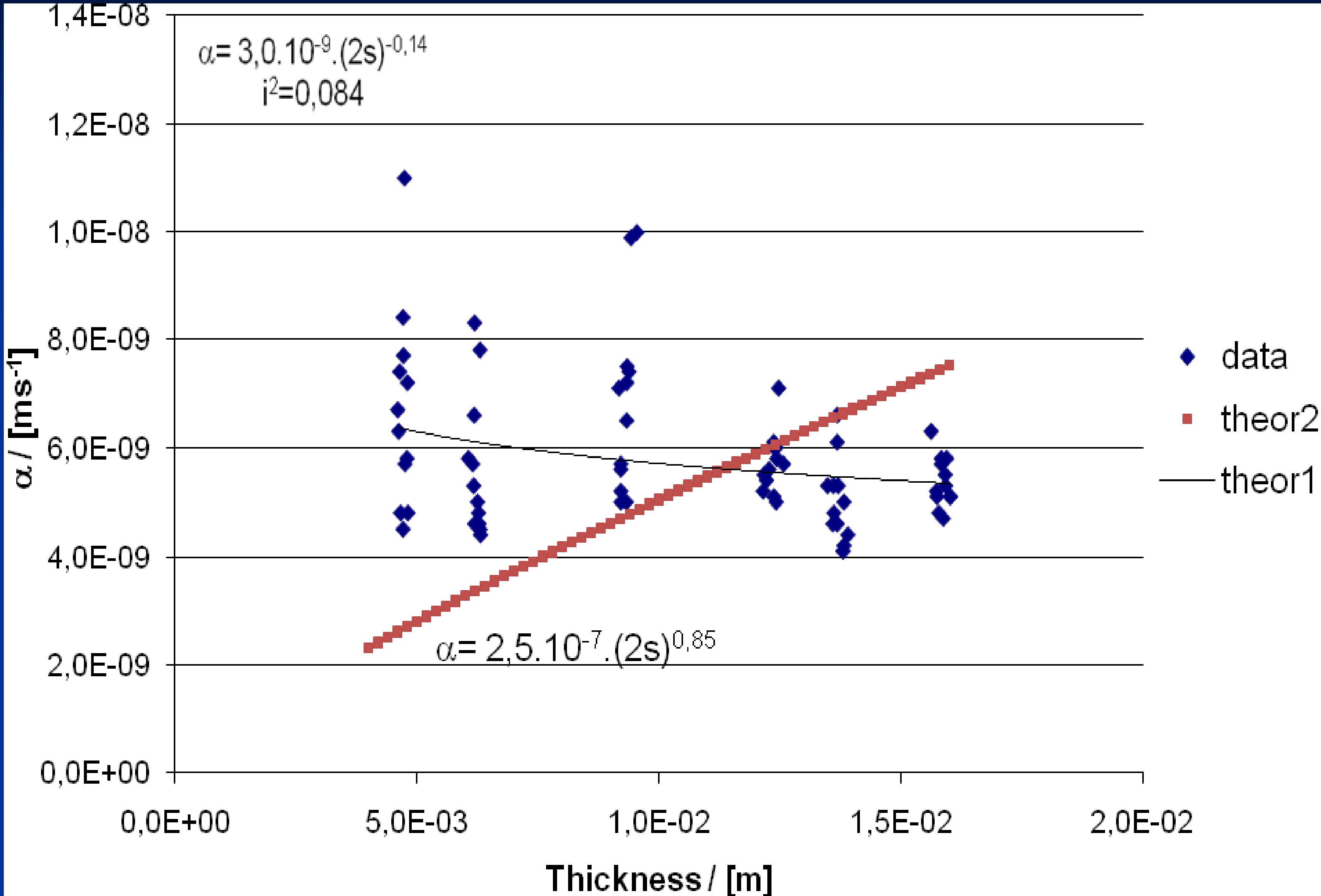
# Results

| Quantity | Statistics | | | | | | |
|---|---|---|---|---|---|---|---|
| 2s | average / [mm] | 15,86 | 13,71 | 12,36 | 9,32 | 6,23 | 4,72 |
| | var. coef. / [%] | 0,75 | 0,88 | 1,02 | 1,17 | 1,23 | 1,49 |
| D | average.$10^{11}$/ [$m^2s^{-1}$] | 1,8 | 1,5 | 1,2 | 0,56 | 0,32 | 0,20 |
| | var. coef. / [%] | 9,4 | 11,8 | 6,9 | 5,2 | 5,5 | 4,8 |
| α | average.$10^9$/ [$ms^{-1}$] | 5,4 | 5,0 | 5,7 | 6,84 | 5,62 | 5,87 |
| | var. coef. / [%] | 8,7 | 14,9 | 10,0 | 25,2 | 23,3 | 33,2 |

# Dependence of diffusion coefficient on thickness:



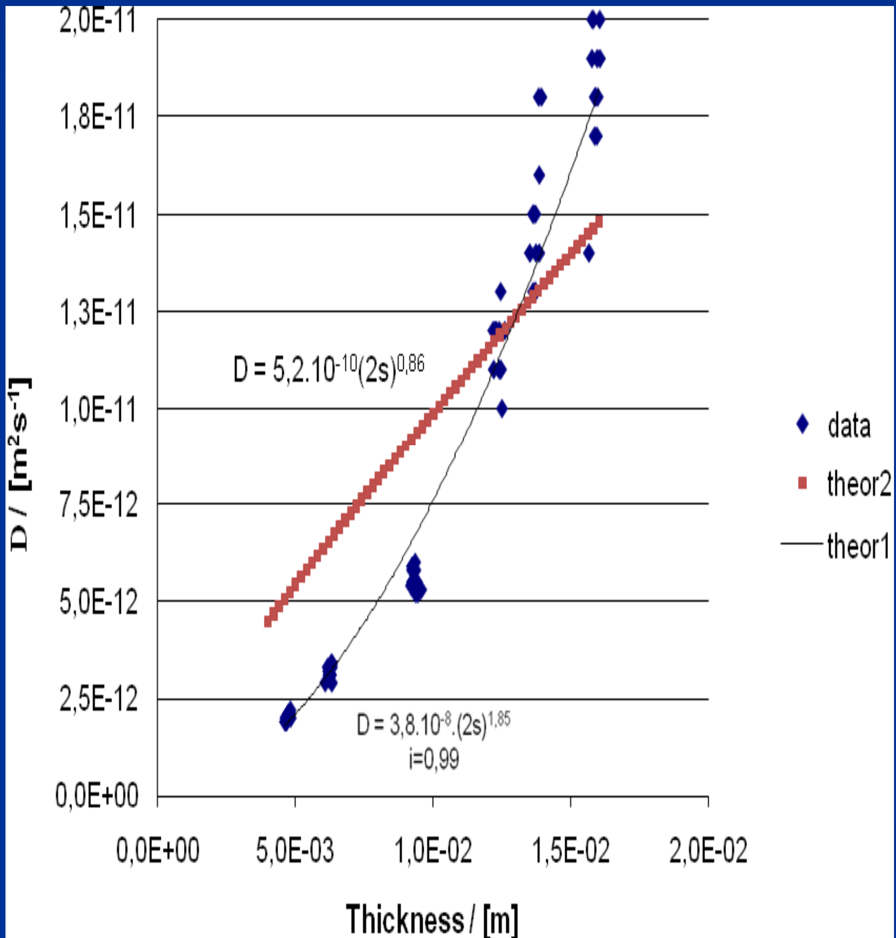
# Dependence of surface emission coefficient on thickness:

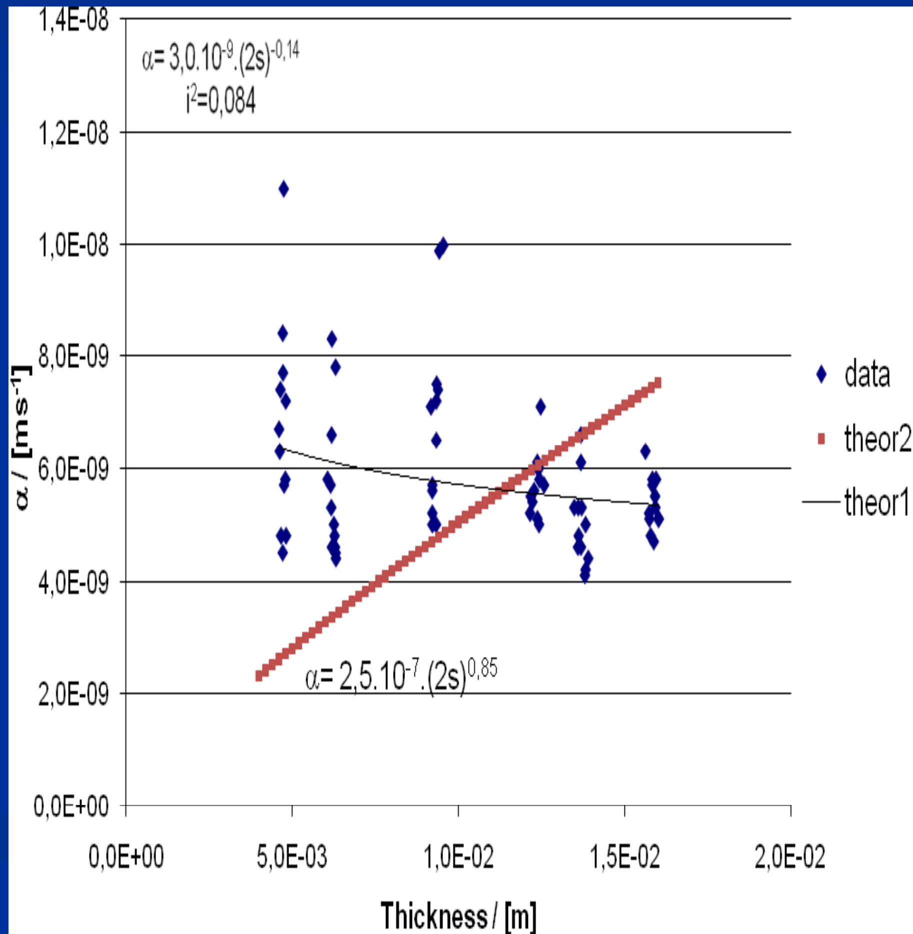


From equality of Fourier numbers and then Biot numbers:

$$\frac{t_1}{t_2} = \left(\frac{s_1}{s_2}\right)^{0,15}$$

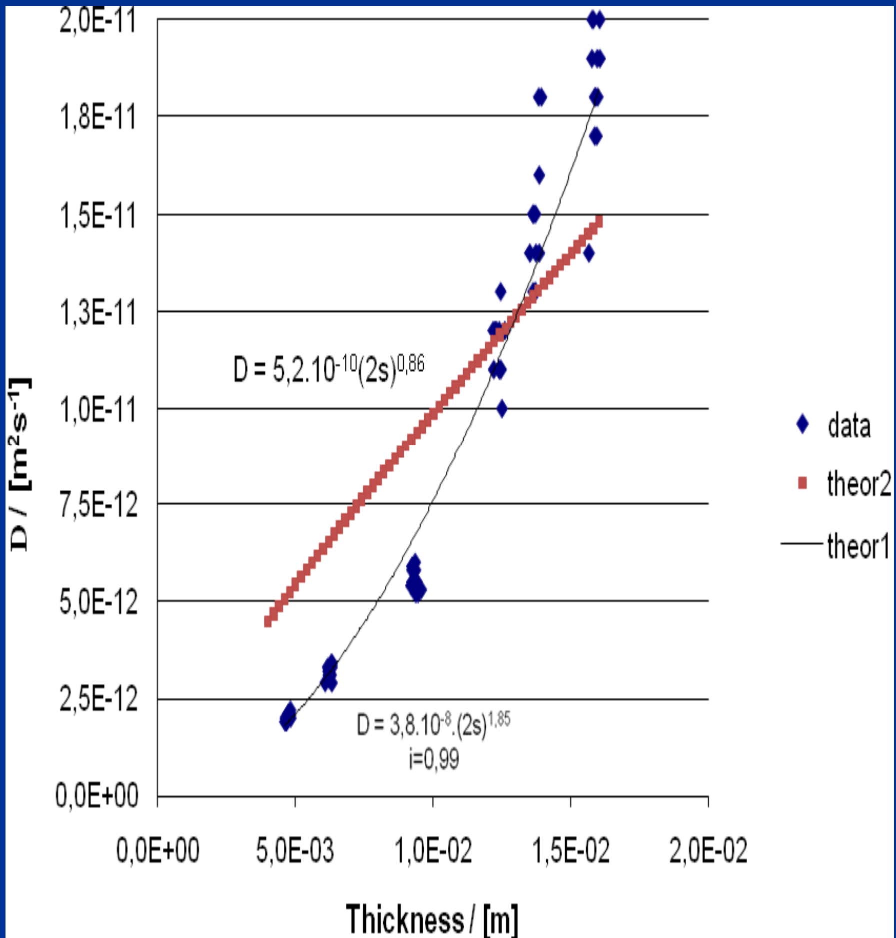
$$\frac{\alpha_1}{\alpha_2} = \left(\frac{s_1}{s_2}\right)^{0,85}$$

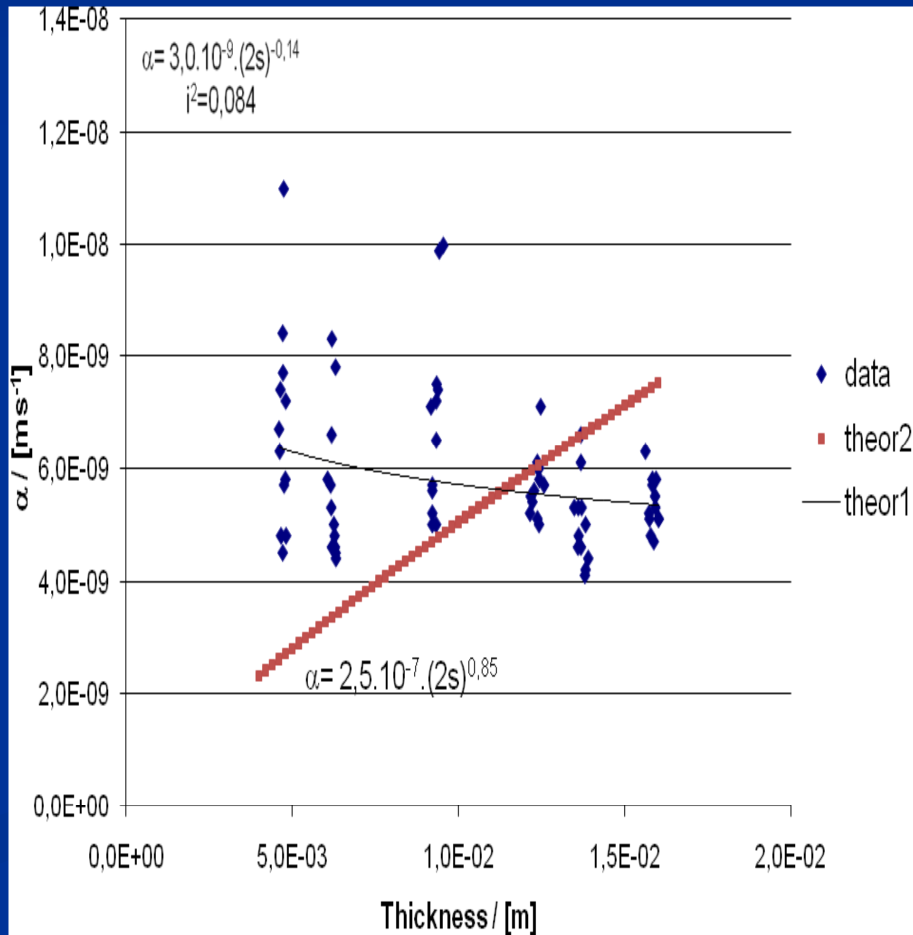


From equality of Biot numbers and then Fourier numbers:

$$\frac{\alpha_1}{\alpha_2} = \left(\frac{s_1}{s_2}\right)^{-0,14} \qquad \frac{D_1}{D_2} = \left(\frac{s_1}{s_2}\right)^{0,86} \qquad \frac{t_1}{t_2} = \left(\frac{s_1}{s_2}\right)^{1,14}$$



# Conclusions

On the basis of experimental data we can conclude:

- The diffusion coefficient is substantially lesser variable quantity then surface emission coefficient.

- The diffusion coefficient increases more slowly than the square power of thickness; therefore the time of diffusion is increasing with the thickness.
  This conclusion was reached after comparing of Biot numbers for specimens of different thicknesses.

These results can be used in the investigation of the relationship wood-water.